Notes 4

CHAPTER 5: THE TIME VALUE OF MONEY, ANNUITY & PERPETUITY

Annuity and Perpetuity

***Definition* -** Perpetuity

A stream of level cash payments that never ends

***Definition* -** Annuity

Level stream of cash flows at regular intervals with a finite maturity

Perpetuity

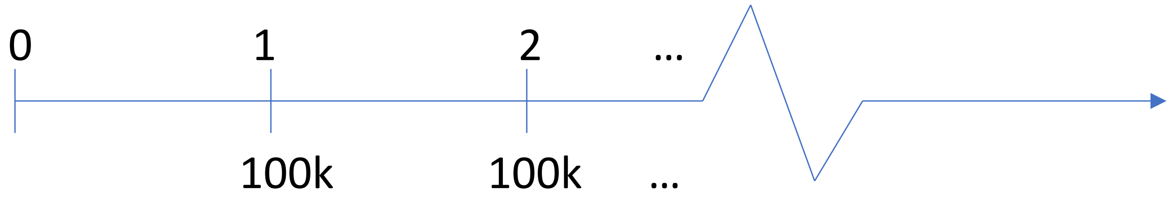
- PV of Perpetuity Formula

$$PV_0 = \frac{C}{r}$$

  - C = cash payment
  - r = interest rate

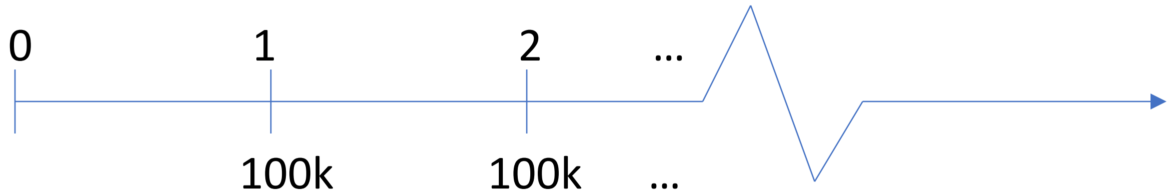
Perpetuity– Example 1

- *In order to create an endowment, which pays $100,000 per year forever, how much money must be set aside today in the rate of interest is 10%?*
- The timeline is as follow:



Perpetuity– Example 1

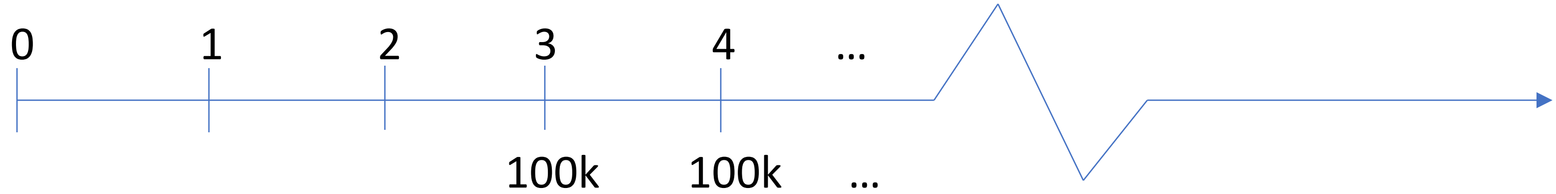
- The timeline is as follow:



- $PV_0 = \frac{C}{r} = \frac{\$100k}{0.1} = \$1{,}000k = \$1\ million$

Perpetuity– Example 2

- *If the first perpetuity payment will not be received until three years from today, how much money needs to be set aside today?*
- The timeline is as follow:



$$PV_2 = \frac{C}{r} = \frac{\$100k}{0.1} = \$1{,}000k = \$1\ million$$

$$PV_0 = \frac{PV_2}{(1+r)^2} = \frac{\$1{,}000k}{1.1^2} = \$826.45k$$

Annuity

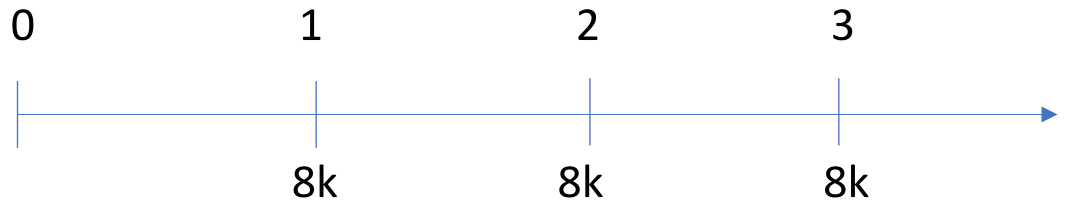
- PV of annuity formula:

$$PV_0 = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

  - C = cash payment
  - r = interest rate
  - t = Number of years cash payment is received

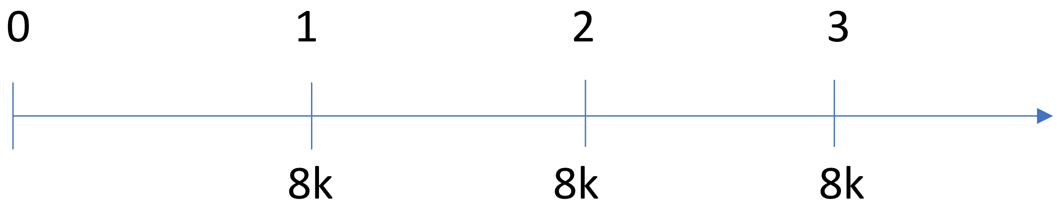
Annuity - Example

- *You are purchasing a car. You are scheduled to make 3 annual installments of $8,000 per year. Given a rate of interest of 10%, what is the price you are paying for the car (i.e., what is the PV)?*
- The timeline is as follow:



Annuity - Example

- The timeline is as follow:

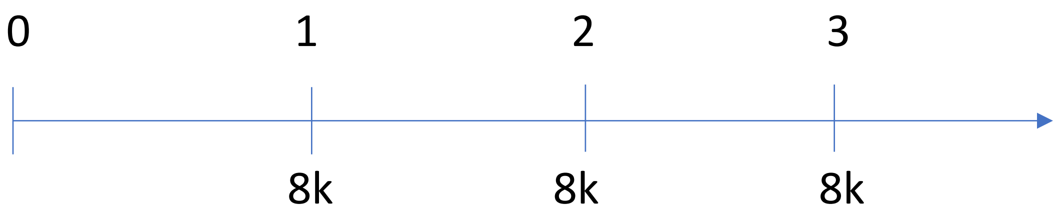


- $PV_0 = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$

$$= 8{,}000\left[\frac{1}{0.1} - \frac{1}{0.1\times(1+0.1)^3}\right] = \$19{,}894.82$$

Annuity - Example

- The timeline is as follow:



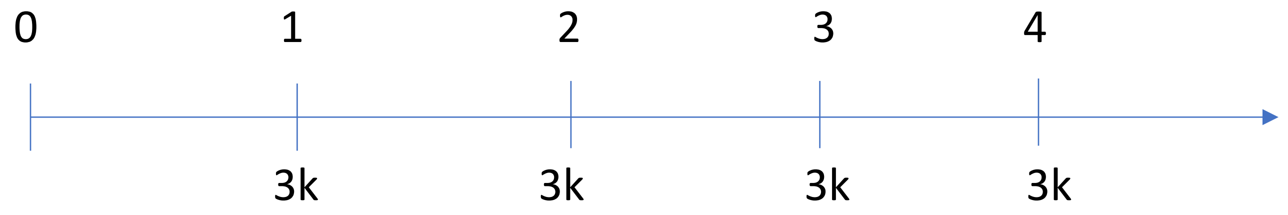
- $PV_0 = \frac{8{,}000}{(1+0.1)^1} + \frac{8{,}000}{(1+0.1)^2} + \frac{8{,}000}{(1+0.1)^3} =$ $\$19{,}894.82$

Annuity and Perpetuity : Applications

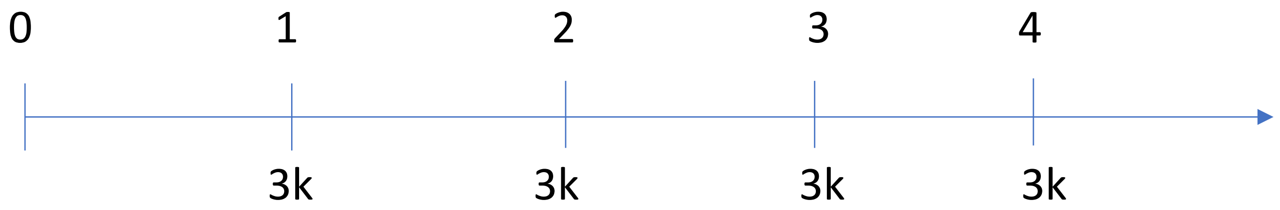
- Value of payments
- Implied interest rate for an annuity
- Calculation of periodic payments
  - Mortgage payment
  - Annual income from an investment payout
  - Future Value of annual payments

Annuity Applications: Future value of annual payments

- You plan to save $3,000 every year for 4 years. Given an 8% rate of interest, what will be the FV of your account?



Annuity Applications: Future value of annual payments



- $PV_0 = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$

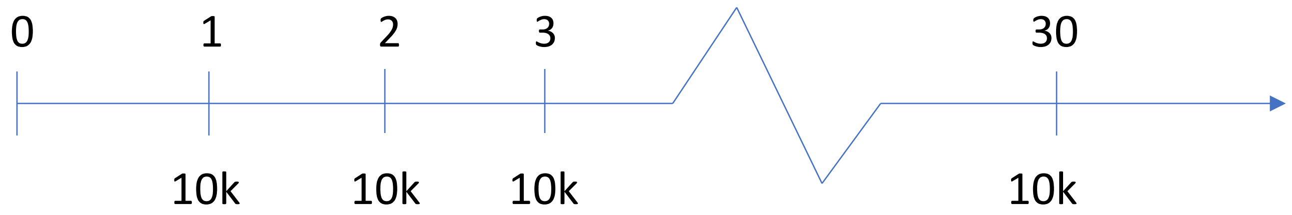
$$= 3{,}000\left[\frac{1}{0.08} - \frac{1}{0.08 \times (1+0.08)^4}\right] = \$9{,}936$$

- $FV_4 = C(1+r)^4 = 9{,}936(1+0.08)^4 = \$13{,}518$

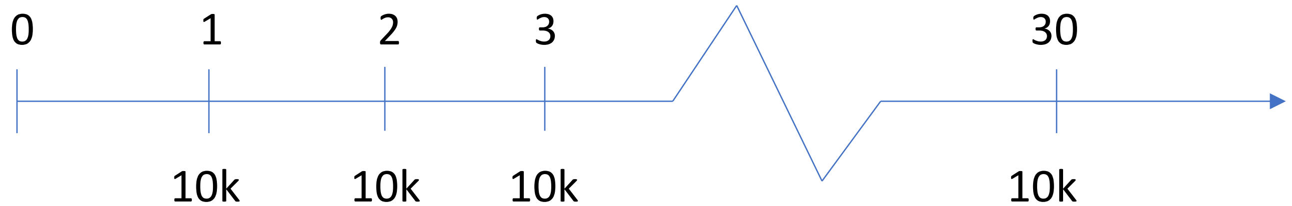
- Again you could have moved each payment ($3,000) to period 4.

Annuity Applications: Loan amount

- *You are purchasing a home and are scheduled to make 30 annual installments of $10,000 per year. Given an interest rate of 5%, what is the price you are paying for the house (i.e. what is the present value)?*



Annuity Applications: Loan amount



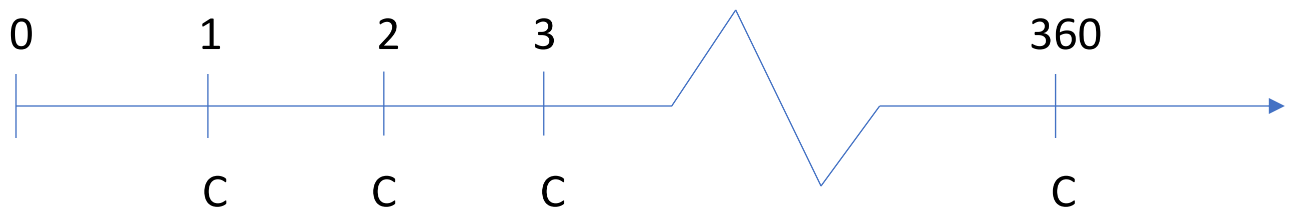
- $PV_0 = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$

$$= 10{,}000\left[\frac{1}{0.05} - \frac{1}{0.05 \times (1+0.05)^{30}}\right]$$

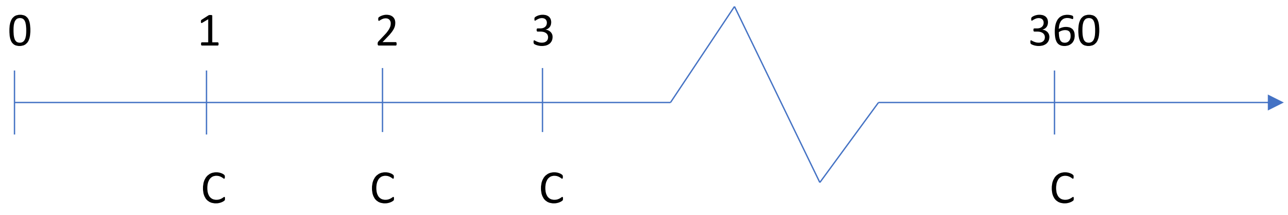
$$= \$153{,}724.51$$

Annuity Applications: Loan Payment

*You are purchasing a home. The home price is $250,000. You have $50,000 in down payment, and you would like to borrow the rest. The mortgage is set for 30 years with monthly payments (360 payments in total). Given **a monthly interest rate of 1%,** what is the monthly payments?*



Annuity Applications: Loan Payment



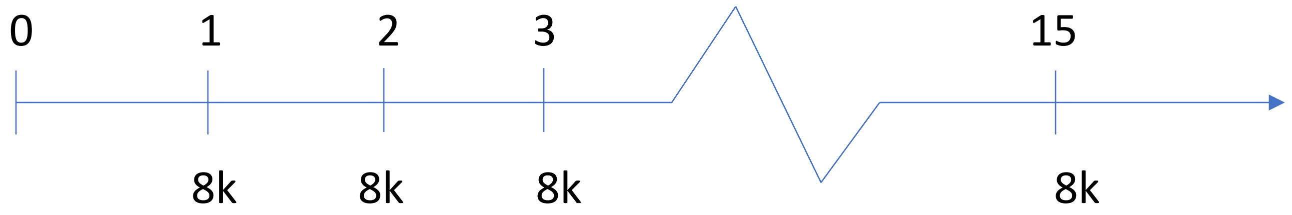
- $PV_0 = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$

- $200{,}000 = C\left[\frac{1}{0.01} - \frac{1}{0.01(1+0.01)^{360}}\right]$

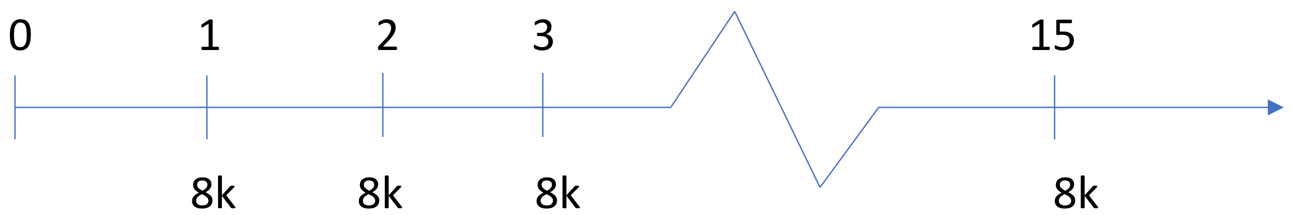
$$C = \frac{200{,}000}{\left[\frac{1}{0.01} - \frac{1}{0.01(1+0.01)^{360}}\right]} = \$2{,}057.23$$

Annuity Applications: Loan Payment

You bought an annuity for $100,000. The annuity pays you $8,000 annually for 15 years. If the first payment is a year from today, what is the implied annual interest rate?

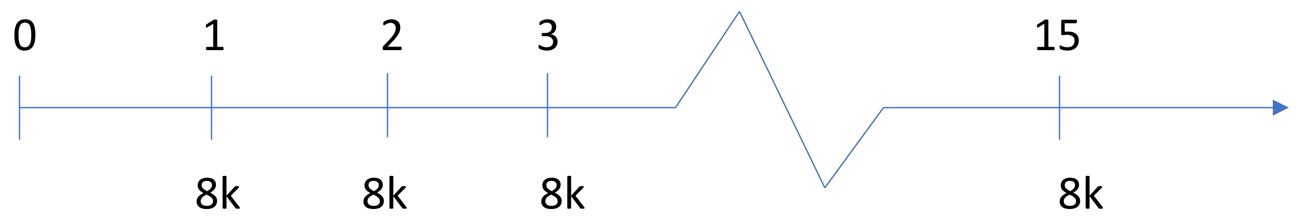


Annuity Applications: Loan Payment



- $PV_0 = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$
- $100k = 10k\left[\frac{1}{r} - \frac{1}{r(1+r)^{15}}\right]$
- $10 = \left[\frac{1}{r} - \frac{1}{r(1+r)^{15}}\right]$
- Next you need to guess the interest rate r=2.37%.

Annuity Applications: Loan Payment

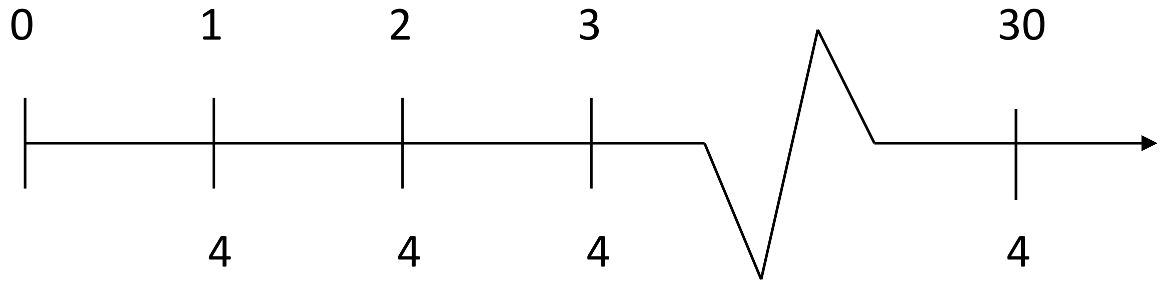


- It’s much easier to calculate the interest rate using Excel. The function is:

=RATE(15,-8,100)

Comparing Annuity to a Lump Sum Payment

- Suppose that you won the lottery. The jackpot is *$120 million* to be paid in *30* equal annual installments starting in period 1 (i.e., a year from today).
- That means that the cash flows looks like:



- If *r=5%,* what is the present value of your winnings?

Comparing Annuity to a Lump Sum Payment

- We can plug it into the annuity formula to get:

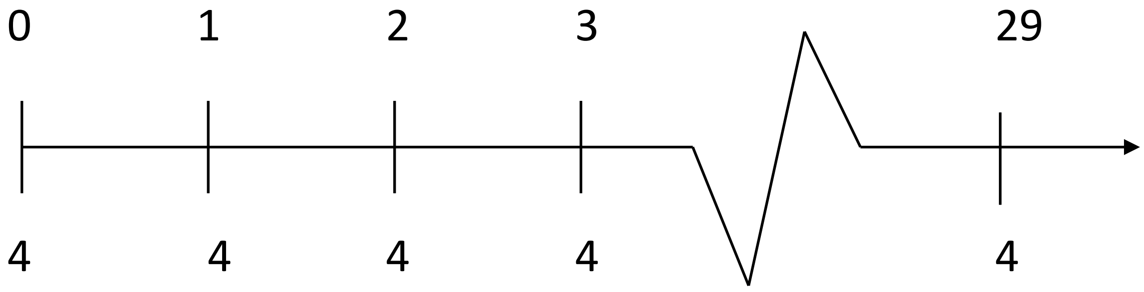
$$PV = 4\left[\frac{1}{0.05} - \frac{1}{0.05 \times 1.05^{30}}\right]$$

$$= \$61.49\ million.$$

- Would you accept a lump sum of $60 million instead of the 30 installment of $4 million?
- Would you accept a lump sum of $65 million?

Annuity Due

- Lets go to the original scenario when the jackpot is $120 million to be paid in 30 annual installments, but now lets assume that the first payment starts today.
- This is called an annuity due and the cash flows looks like:



Annuity Due

- Note that the last 29 payments (from period 1 to 29) is an annuity with t=29. Therefore, we can calculate the present value of the last 29 payments using the annuity formula and add the first payment (in period 0) to get the present value of all the 30 payments:

- $PV = 4 + 4\left[\frac{1}{0.05} - \frac{1}{0.05 \times 1.05^{29}}\right]$=$64.56 million.