Muhlenberg College

Notes 3

CHAPTER 5: THE TIME VALUE OF MONEY, COMPOUND INTEREST

Introduction

- Suppose that I offer to give you $100. Which of the following two options would you prefer and why?
  1. $100 today
  2. $100 a year from today

Introduction

- Three reason for why people prefer the $100 today option:
  1. They can earn interest (from saving account) on money today.
  2. Prices go up all the time (inflation). Purchasing power is better with $100 today.
  3. Preferences. I just like to get money in the present more than in the future.

The Time Value of Money

Definition - Future Value (FV)

Amount to which an investment will grow after earning interest

Definition - Compound Interest

Interest earned on interest

Definition - Simple Interest

Interest earned only on the original investment

Example – Future Value with Simple Interest Rate

- Interest earned at a rate of 6% for five years on a principal balance of $100
- The interest earned each year is

*$100*6% = $6.*

- Interest earned at a rate of 6% for five years on a principal balance of $100
- Value at the end of year 5: ***$130.***

Example – Future Value with Compound Interest Rate

- Interest earned at a rate of 6% for five years on the previous year's balance
- Value at the end of year 5:

$$\boldsymbol{FV_5 = \$133.82.}$$

Future Value

- Future value of $C (C stands for cash):

$$FV_t = C\,(1 + r\,)^t$$

- **For Example**: What is the value of $150 in 7 years if the interest rate is *r = 10%?*

$$FV_7 = 150\,(1 + 0.1)7 = \$292.31.$$

- What is the future value if the interest rate is simple?
- Note that you should always assume compound interest unless the question says “simple interest"

Present Value (PV)

***Definition* - Present Value (PV)**

Value today of a future cash

- The formula for an amount C_t received in t periods (such as years) is:

$$PV_0 = \frac{C_t}{(1+r)^t}$$

Present Value – Example 1

- What is the present value of $100 five years from today if the interest rate is 10%?

- $PV_0 = \frac{C_t}{(1+r)^t} = \frac{\$100}{(1+0.1)^5} = \$62.09$

Present Value – Example 2

- You just bought a new computer for $3,000. The payment terms are 2 years same as cash. If you can earn 8% on your money, how much money should you set aside today in order to make the payment when due in two years?

- Answer:

$$PV_0 = \frac{C_t}{(1+r)^t} = \frac{\$3{,}000}{(1+0.08)^2} = \$2{,}572$$

Present Value – Example 3

- Italy borrowed money for 2 years, but it did not announce an interest rate. It simply offered to sell each €1,000 IOU (promissory note) for € 963.06. What is the interest rate?

- Answer:

$$PV_0 = \frac{C_t}{(1+r)^t}$$

$$963.06 = \frac{€1{,}000}{(1+r)^2}$$

r=1.9%.

Present Value – Example 4

- Your bank offers two different saving accounts for two years with no exits allowed:

  1. First year interest=20% and second year interest=5%.
  2. First year interest=5% and second year interest=20%.

  Which program (if any) is better?

Multiple Cash Flows and Timeline

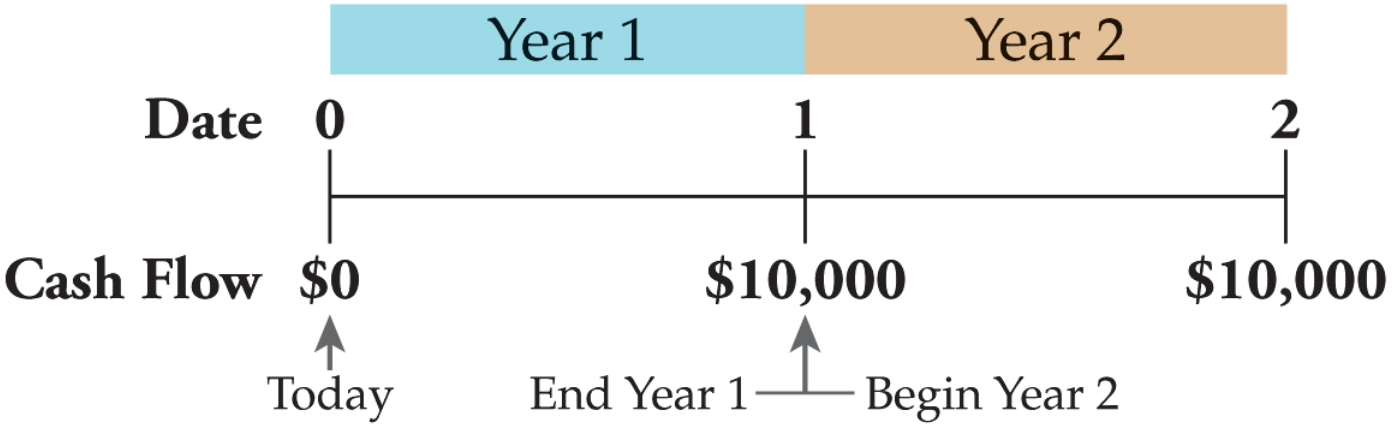
***Definition* -** Timeline

A timeline is a linear representation of the timing of potential cash flows.

- Drawing a timeline of the cash flows will help you visualize the financial problem.

Multiple Cash Flows and Timeline

- Assume that you made a loan to a friend. You will be repaid in two payments, one at the end of each year over the next two years.



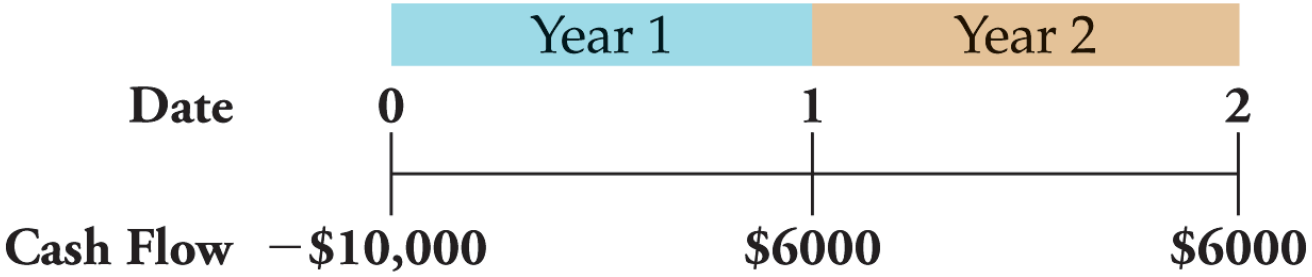
Multiple Cash Flows and Timeline

Differentiate between two types of cash flows

- Inflows are positive cash flows.
- Outflows are negative cash flows, which are indicated with a – (minus) sign.

Multiple Cash Flows and Timeline

- Assume that you are lending $10,000 today and that the loan will be repaid in two annual $6,000 payments.



- The first cash flow at date 0 (today) is represented as a negative sum because it is an outflow.
- Timelines can represent cash flows that take place at the end of any time period – a month, a week, a day, etc.

Multiple Cash Flows and Timeline

- Financial decisions often require combining cash flows or comparing values. Three rules govern these processes.
- The Three Rules of Time Travel
  1. Only values at the same point in time can be compared or combined.
  2. To move a cash flow forward in time, you must compound it using the formula: $FV_t = C\,(1 + r\,)^t$
  3. To move a cash flow backwards in time, you must discount it using the formula:

$$PV_0 = \frac{C_t}{(1+r)^t}$$

Multiple Cash Flows and Timeline - Example

- You are able to put $1,200 in the bank now, and another $1,400 in 1 year. If you earn an 8% rate of interest, how much will you be able to spend on a computer in 2 years?

- The value of $1,400 in year 2:

$$FV_2 = 1{,}400(1 + 0.08)^1 = \$1{,}512$$

- The value of $1,200 in year 2

$$FV_2 = 1{,}200(1 + 0.08)^2 = \$1{,}399.68$$

- Together the value of the two deposits in year 2

$$FV_2 = 1{,}512 + 1{,}399.68 = 2{,}911.68$$

PV of Multiple Cash Flows

- PVs can be added together to evaluate multiple cash

$$PV_0 = C_0 + \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \cdots$$

PV of Multiple Cash Flows - Example

- Your auto dealer gives you the choice to pay $15,500 cash now, or make three payments: $8,000 now and $4,000 at the end of the following two years. If your cost of money is 8%, which do you prefer?
- First draw the timeline!
- Then calculate the PV as:

$$PV_0 = 8{,}000 + \frac{4{,}000}{(1+0.08)^1} + \frac{4{,}000}{(1+0.08)^2} = \$15{,}133.1$$

- Since $15,133.1 <$15,500, it is better to do the three payments.

PV of Multiple Cash Flows - Example